

Simple parametric analysis for a multi-state model in hospital epidemiology

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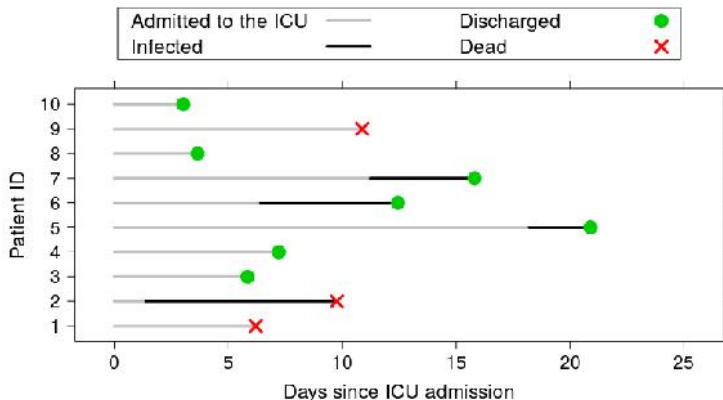
- | **Key message:**

- | When analysing a new dataset try to start out simple

- | **Questions to discuss during the day:**

- | How do others bridge the gap between simplification and misspecification?
- | How do others bridge the gap between correct (but complex) modelling and interpretability?

Motivating example: Sample of the SIR-3 study



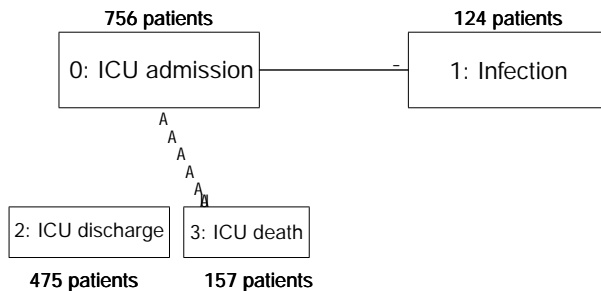
- Observational cohort study from the Charite university hospital in Berlin Germany on hospital-acquired pneumonia in intensive care
- Investigate burden of hospital-acquired infection

Motivating example: Sample of the SIR-3 study



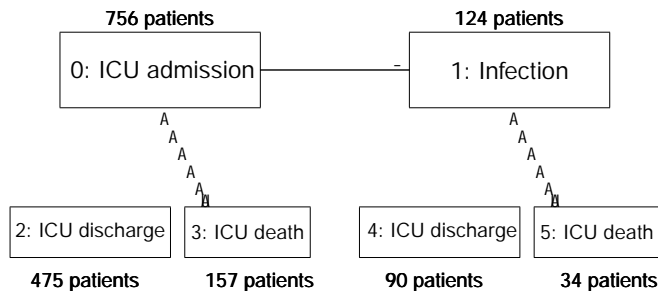
- | Commonly used: Kaplan-Meier estimator for classical survival-model

Motivating example: Sample of the SIR-3 study



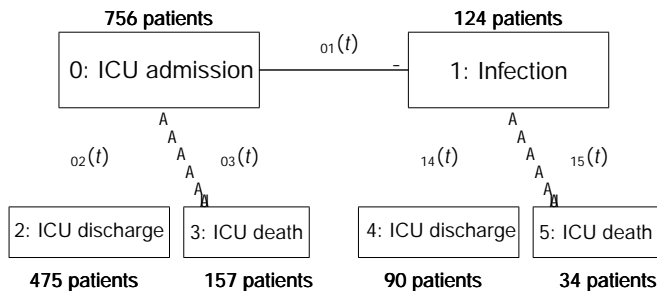
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Motivating example: Sample of the SIR-3 study



- | Commonly used: Kaplan-Meier estimator for classical survival-model
- | Source of bias! Kaplan-Meier estimator treats competing events as non-informative
- | Watch out for: Competing risks bias, time-dependent bias

Motivating example: Sample of the SIR-3 study



- | Transition specific hazard rates $\lambda_{ij}(t); i, j \in \{0, 1, 2, 3, 4, 5\}$
- | Transition probabilities $P_{ij}(s; t)$
- | Hazard ratios of infection, relative risk of death

Parametric estimation: Assuming constant hazards

- | Assume $\lambda_{ij}(t) = \lambda_{ij}$ for all possible transitions
- | Simple summary analysis

Occurrence of infection	
Incidence rate per day	01

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Occurrence of infection	
Incidence rate per day	λ_{01}
Risk of infection	$\lambda_{01} = (\lambda_{01} + \lambda_{02} + \lambda_{03})$

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Occurrence of infection

Incidence rate per day

λ_{01}

Risk of infection

$\lambda_{01} = (\lambda_{01} + \lambda_{02} + \lambda_{03})$

Mortality

Death hazard ratio of infection

$\lambda_{15} = \lambda_{03}$

Discharge hazard ratio of infection

$\lambda_{14} = \lambda_{02}$

Parametric estimation: Assuming constant hazards

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Occurrence of infection

Incidence rate per day	λ_{01}
Risk of infection	$\lambda_{01} = (\lambda_{01} + \lambda_{02} + \lambda_{03})$
Mortality	
Death hazard ratio of infection	$\lambda_{15} = \lambda_{03}$
Discharge hazard ratio of infection	$\lambda_{14} = \lambda_{02}$
ICU mortality risk of infected patients	$\lambda_{15} = (\lambda_{14} + \lambda_{15})$
ICU mortality risk of uninfected patients	$\lambda_{03} = (\lambda_{02} + \lambda_{03})$
ICU mortality risk ratio	$\lambda_{15} = \lambda_{03} \cdot (\lambda_{02} + \lambda_{03}) = (\lambda_{14} + \lambda_{15})$

Parametric estimation: Assuming constant hazards

- | Other quantities such as odds of infection/ICU death, extra length of hospital stay
- | Even the transition probabilities of the extended illness-death model have closed mathematical forms!
- | Example: $P_{ij}(s; t) = P_{ij}(0; t)$, $t = t - s$
 - | $P_{00}(s; t) = P_{00}(0; t) = \exp(-(\lambda_{01} + \lambda_{02} + \lambda_{03})t)$
 - | $P_{03}(s; t) = P_{03}(0; t) = \frac{\lambda_{03}}{\lambda_{01} + \lambda_{02} + \lambda_{03}} (1 - \exp(-(\lambda_{01} + \lambda_{02} + \lambda_{03})t))$

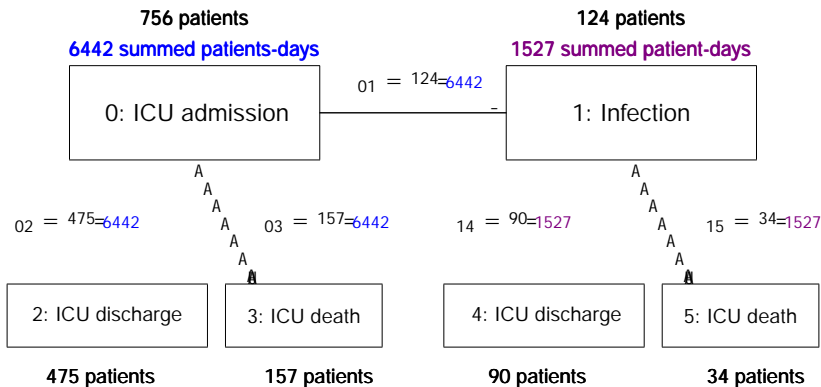
Parametric estimation: Assuming constant hazards

- | Estimate constant hazard rates with maximum likelihood estimator:

$$\hat{\lambda}_{ij} = \frac{\text{Number of } i \rightarrow j \text{ transitions}}{\text{Summed patient-days in state } i}$$

- | Plug into the formulas
 -) **No sophisticated software is necessary for a basic multi-state analysis**
- | All you need:
 - | Summed patient days in state 0 and 1
 - | Number of patients in each state at end of follow-up

Applied to the sample of the SIR-3 study

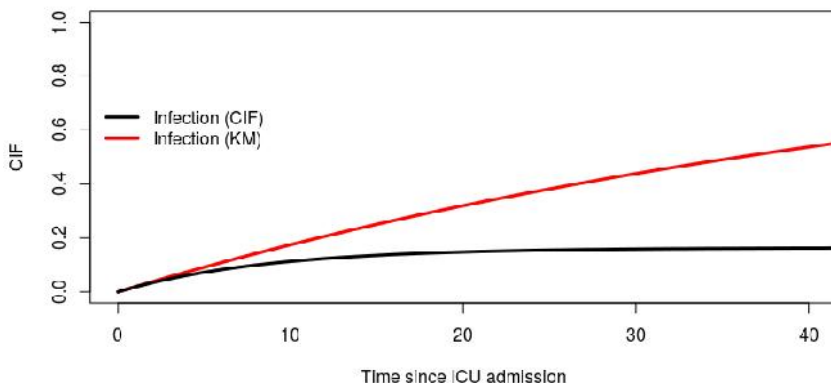


Data example: Summary analysis of sample of SIR-3 study

Occurrence of infection	
Incidence rate per day	0.019
Risk of infection	16.4%
Mortality	
Death hazard ratio of infection	0.91
Discharge hazard ratio of infection	0.80
ICU mortality risk of infected patients	27.4%
ICU mortality risk of uninfected patients	24.8%
ICU mortality risk ratio	1.11

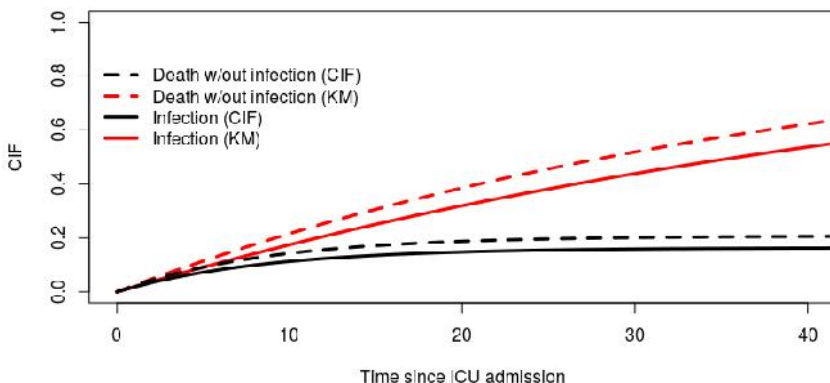
Parametric cumulative incidence functions to demonstrate competing-risks bias

**Cumulative incidence functions (CIFs) in state 0,
the red lines are the according Kaplan-Meier estimators**



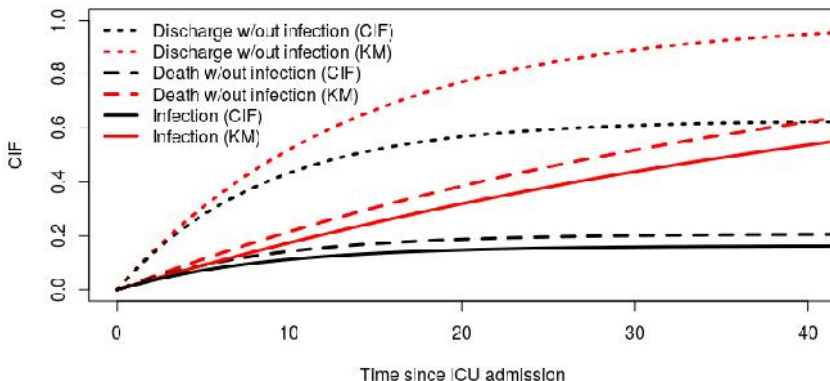
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- | Competing risks bias, time-dependent bias are avoided
- | Method can be used for reanalysis of published data and meta analysis
 -) In publications: always provide mean length of stay spent in the transient states and the number of observed events!

Summary

- | Basic multi-state analysis can be performed by assuming constant hazards
 -) Provides first insights into data structure and time-dynamics
- | Competing risks bias, time-dependent bias are avoided
- | Method can be used for reanalysis of published data and meta analysis
 -) In publications: always provide mean length of stay spent in the transient states and the number of observed events!
- | Method applicable to any other data setting with competing risks and binary exposure

Conclusion



Start simple,

Conclusion



Start simple,
then increase complexity

References

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Non-parametric estimation in multi-state models

- | Nelson-Aalen estimator of cumulative transition-specific hazards
R-package: mvna

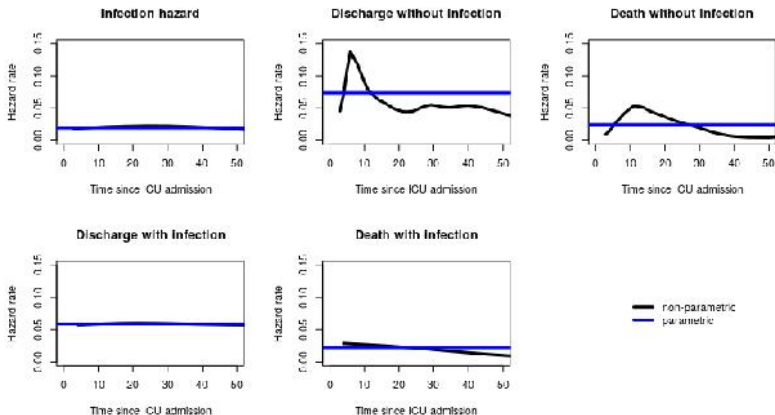
- | Aalen-Johansen estimator of transition-probabilities
R-package: etm, mstate

- | Smoothing methods can be used to obtain the transition-specific hazard rates
R-package: bshazard

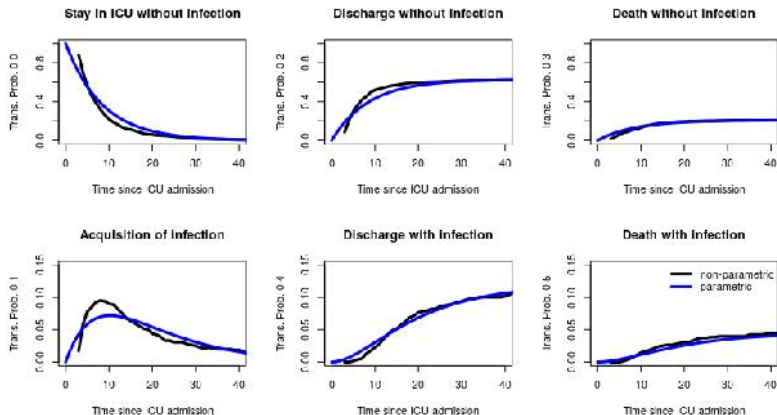
- | Cox proportional hazards model to estimate hazard ratios
R-package: survival

-) For more details and application see "Competing risks and Multistate models with R" by Beyersmann et al. (2011)

Misspecification due to false assumption of constant hazards?



Parametric and non-parametric transition probabilities from state 0



Data example: Summary analysis of sample of SIR-3 study

Occurrence of infection / length of stay due to infection	
Incidence rate per day	0.019
Risk of infection	16.4%
Change in length of stay due to infection (cLOS)	1.77 days

| cLOS in naive approach (time-dependent bias):

$$\begin{aligned}
 (\text{?LOS of infected patients}) \quad (\text{?LOS of uninfected patients}) &= \\
 21:0 \quad 8:5 &= 12:5
 \end{aligned}$$